Advanced signal processing based on support vector regression for LIDAR applications

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ABSTRACT

The LIDAR technique has recently found many applications in atmospheric physics and remote sensing. One of the main issues, in the deployment of systems based on LIDAR, is the filtering of the backscattered signal to alleviate the problems generated by noise. Improvement in the signal to noise ratio is typically achieved by averaging a quite large number (of the order of hundreds) of successive laser pulses. This approach can be effective but presents significant limitations. First of all, it implies a great stress on the laser source, particularly in the case of systems for automatic monitoring of large areas for long periods. Secondly, this solution can become difficult to implement in applications characterised by rapid variations of the atmosphere, for example in the case of pollutant emissions, or by abrupt changes in the noise. In this contribution, a new method for the software filtering and denoising of LIDAR signals is presented. The technique is based on support vector regression. The proposed new method is insensitive to the statistics of the noise and is therefore fully general and quite robust. The developed numerical tool has been systematically compared with the most powerful techniques available, using both synthetic and experimental data. Its performances have been tested for various statistical distributions of the noise and also for other disturbances of the acquired signal such as outliers. The competitive advantages of the proposed method are fully documented. The potential of the proposed approach to widen the capability of the LIDAR technique, particularly in the detection of widespread smoke, is discussed in detail.

Keywords: Lidar, Wild fires, Widespread smoke, Support Vector Regression, Robust smoothing

1. INTRODUCTION: LIDAR MEASUREMENTS OF PARTICULATE

Lidar measurements have become well established laser based techniques for remote sensing of the atmosphere [1]. They are used to probe almost any altitude in the most different conditions, from forests to urban areas. One of the most interesting applications consists of environment surveying of particulate [2-10]. Indeed, with the development of reliable lasers, emitting in the appropriate range of wavelengths, Lidar systems are suitable and competitive techniques [11, 12]. A typical example of the use of Lidar is the detection of forest fires [3, 4, 6, 11]. Indeed wild fires have become a very serious problem in various parts of the world. The LIDAR technique has been successfully applied to the detection of the smoke plume emitted by wild fires, allowing the reliable survey of large areas. The main operational approach envisages the continuous monitoring of the area to be surveyed with a suitable laser. When a significant peak in the backscattered signal is detected, an alarm is triggered. The traditional applications of Lidar systems to atmospheric physics therefore rely on the capability of properly detecting the backscattered peaks of radiation. More recently, the LIDAR technique has been shown to have the potential to provide useful measurements also of widespread smoke, which can be the consequence of strong wind dispersion or non-concentrated sources [13]. Typical examples of backscattered signals for the alternatives of clear atmosphere, strong smoke plume and widespread smoke are shown in Figure 1.
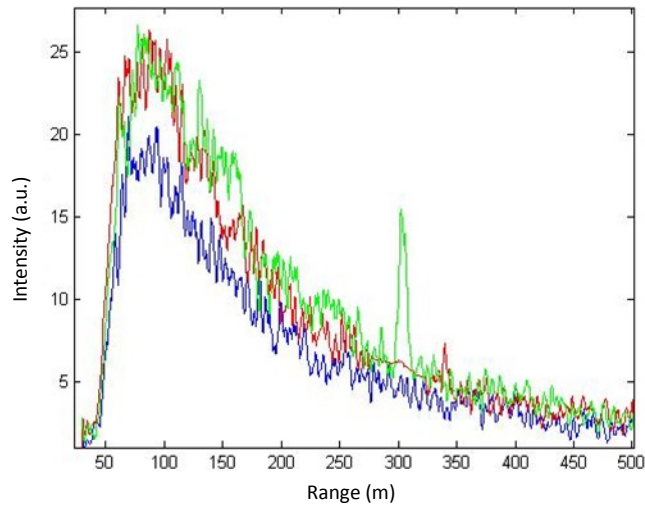
Figure 1. Examples of LIDAR back scattered signals: a) Clear atmosphere (blue line) b) strong smoke plume (green line) c) widespread smoke (red line).

As can be seen from the experimental signals shown in Figure 1, strong localized emissions of particulates produce short peaks in the LIDAR backscattering. Larger, less concentrated and more wide spread sources of particulate result in an increase of the entire curve. This can be ascribed to an increase in the backscattering coefficient over practically the entire range of the LIDAR system

In all these applications a lot of the data analysis is preformed manually, which is acceptable for pioneering tests and even experimental campaigns of the order of days. Such an approach is of course not no viable for systematic applications. In this perspective, new and automatic analysis techniques can help substantially, when results of very large surveys have to be analyzed or when real time alarms have to be reliably guaranteed. An original tool, the Universal Multi-Event Locator (UMEL) [14, 15], has already been applied successfully to the problem of automatically identifying the time location of peaks in the backscattered LIDAR signals. The method developed is based on Support Vector Regression (SVR) and presents various advantages with respect to more traditional techniques. In this paper, Support Vector Regression is used to improve both smoothing and modeling of the backscattered signals. The added value of SVR consists of an increased robustness against noise, which can have significant practical consequences. SVR can be used as a way to clean the backscattered signals in order to allow more reliable fitting. This method can also help in increasing the time resolution of the measurements and in widening the applicability of the technique.

With regard to the structure of the paper, in the next Section 2 the mathematical background on Support Vector Regression is provided. Section 3 reports a series of numerical tests aimed at showing the potential of the proposed method of signal processing based on SVR. An overview of the Lidar system used to perform the experimental measurements analyzed in the rest of the paper is given in Section 4. The application of the new signal processing methods to the experimental measurements are the subject of Section 5. In Section 6 some lines of possible future investigations are provided.

2. SUPPORT VECTOR REGRESSION FOR THE FIRST SIGNAL PROCESSING

Support Vector Machines are a very specific class of machine learning tools, whose characteristics are use of kernels, absence of local minima, sparseness of the solution and generalization control obtained by acting on the margins. They were invented by Vladimir Vapnik and his co-workers, and first introduced at the Computational Learning Theory (COLT) 1992 conference. In 1995 also the soft margin version of SVM was introduced. Originally conceived for

classification, SVM can be applied also to regression. Still they present all the main features that characterize maximum margin algorithms: a non-linear function is learned by mapping the inputs into a high dimensional feature space induced by a suitable kernel. In analogy with classification, there are significant advantages in optimizing the generalization of the regression margins. This is achieved by defining an indicator, called loss function, which ignores errors, whose amplitude is situated within a certain distance of the true value. This type of function is often called – epsilon insensitive – loss function. Figure 2 shows an example of one-dimensional linear regression function with an epsilon insensitive band.
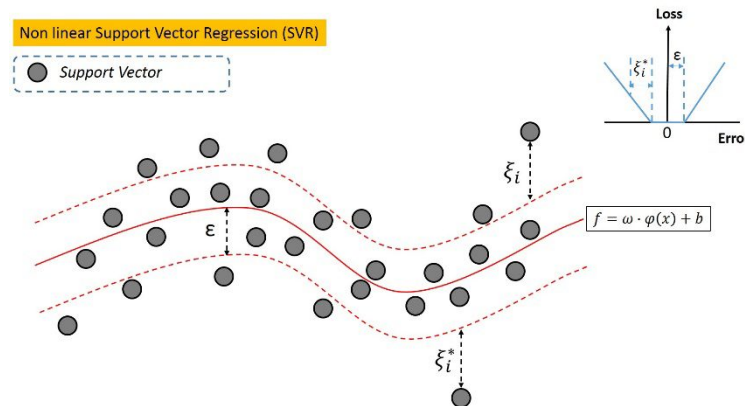

Figure 2. Example of an epsilon insensitive loss function

The loss function quantifies the cost of the errors at the training points. The errors are considered zero for all points that are inside the insensitive band.

In SVM regression, the input x is first mapped onto a m-dimensional feature space, using some fixed (nonlinear) mapping, and then a linear model is constructed in this feature space. Using mathematical notation, the linear model (in the feature space) f(x, ω) is given by:

$$f(x,\omega) = \sum_{j=1}^{m} \omega_j g_j(x) + b \qquad (1)$$

where $g_j(x)$, $j=1,...,m$ denotes a set of nonlinear transformations, and b is the "bias" term. As mentioned, the quality of the estimation is measured by the loss function $L(y, f(x,\omega))$. SVM regression uses a new type of loss function, called ε-insensitive loss function, introduced by Vapnik:

$$L_\varepsilon(y, f(x,\omega)) = \begin{cases} 0 & if\ |y - f(x,\omega)| \le \varepsilon \\ |y - f(x,\omega)| - \varepsilon & otherwise \end{cases} \qquad (2)$$

The so-called empirical risk can be calculated as:

$$R_{emp}(\omega) = \frac{1}{n} \sum_{i=1}^{n} L_\varepsilon(y_i, f(x_i,\omega)) \qquad (3)$$

SVM regression performs linear regression in the high-dimension feature space using ε-insensitive loss and, at the same time, tries to reduce model complexity by minimizing $\|\omega\|^2$. This can be described by introducing (non-negative) slack variables $\xi_i, \xi_i^*\ i = 1,...n$, to measure the deviation of training samples outside the ε-insensitive zone. Thus, SVM regression is formulated as minimization of the following functional:

$$\min \quad \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{n}\left(\xi_i + \xi_i^*\right) \qquad (4)$$

with the constraints

$$\begin{cases} y_i - f(x_i, \omega) \leq \varepsilon + \xi_i^* \\ f(x_i, \omega) - y_i \leq \varepsilon + \xi_i \\ \xi_i, \xi_i^* \geq 0,\ i = 1, \ldots, n \end{cases} \quad (5)$$

This optimization problem is solved by constructing a lagrangian function using relation (4) and implementing the constraints (6). To improve computability and to extend the space of solutions to nonlinear ones, the lagrangian can be reformulated in terms of dual variables and a suitable kernel. The solution of the resulting optimization problem is given by

$$f(x) = \sum_{i=1}^{n_{SV}}\left(\alpha_i - \alpha_i^*\right)K(x_i, x) \qquad (6)$$

$$0 \leq \alpha_i^* \leq C \quad 0 \leq \alpha_i \leq C$$

where n_{SV} is the number of Support Vectors (SVs) and the kernel function

$$K(x, x_i) = \sum_{j=1}^{m} g_j(x)g_j(x_i) \qquad (7)$$

It is well known that SVM generalization performance (estimation accuracy) depends on a good setting of meta-parameters C, ε and the kernel parameters. Existing software implementations of SVM regression usually treat SVM meta-parameters as user-defined inputs. Selecting a particular kernel type and kernel function parameters is usually based on application-specific knowledge and should reflect the distribution of the input values of the training data. Parameter C determines the tradeoff between the model complexity (smoothness) and the degree to which deviations larger than ε are tolerated. For example, if C is too large (infinity), then the objective is reduced to minimizing the empirical risk only, without constraints on model complexity. Parameter ε controls the width of the ε -insensitive zone, used to fit the training data. The value of ε can affect the number of support vectors used to construct the regression function. The bigger ε, the fewer support vectors are selected. On the other hand, bigger ε-values results in smoother estimates.

3. SUPPORT VECTOR REGRESSION: PERFORMANCE IN FILTERING AND SMOOTHING

To assess the potential of filtering and smoothing with SVR, a series of systematic test have been performed. First, the capability to reduce the number of laser pulses to obtain a certain quality of the data has been verified. Typically, in LIDAR applications denoising is obtained by adding a quite high number of subsequent backscattered signals (typical averaging methodologies). If the noise is random and additive, this operation increase significantly the signal to noise ratio. On the other hand, having to multiply the measurements to obtain high quality data can reduce the time resolution and add stress to the hardware, in particular the laser source. SVR can help significantly in this respect. This can be appreciated by inspection of Figure 3, in which the performance of the traditional approach is compared with filtering using SVR. Synthetic signals have been generated using a lognormal function. The lognormal function is defined as:

$$\mathcal{N}(\ln x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{(\ln x - \mu)^2}{2\sigma^2}\right], \quad x > 0. \quad (8)$$

White noise within a range of ± 10 % of the maximum has then been added to the signals (top plot of Figure 3). The central plot of Figure 3 shows the improvement in the S/N ratio obtained by adding 10 pulses, whereas the bottom plot is a signal obtained by filtering a single backscattered pulse with SVR.
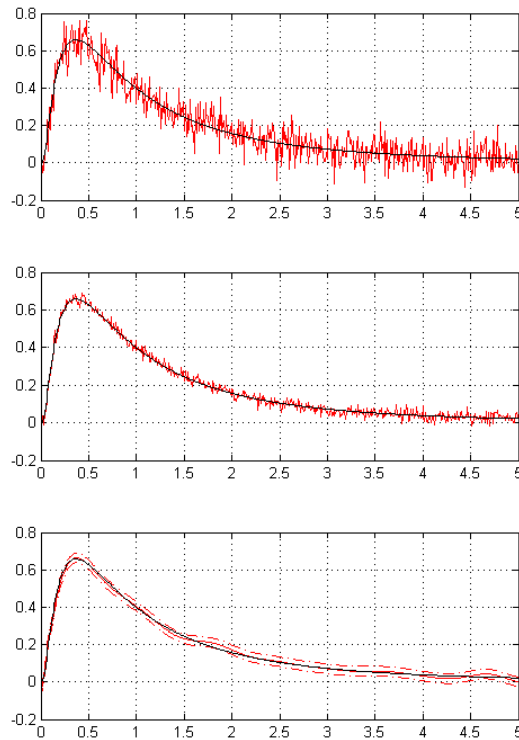

Figure 3 Top: one realization of the lognormal signal including additive noise (Gaussian distribution with σ equal to 10% of the peak value of the signal). Centre: signal obtained summing 10 independent realizations of the signal. Bottom: an individual realization filtered with SVR (the red dashed lines indicate the ε-tube).

In any case, to test more generally the performance of SVR, a comparison with the most consolidated smoothing methods has been performed. In practice the most used filtering techniques are the ones belonging to the following families [17]:

- Moving average
- Lowess
- Loess
- Rlowess: robust version of Lowess
- Rloess: robust version of Lowess
- Savitzky-Golay

The moving average is obtained by calculating a series of averages of different subsets of the full data set. Given a time series and a fixed subset size, the first element of the moving average is calculated by taking the average of the initial fixed subset of points of the series. Then the subset is changed by a process of "shifting forward"; that is, excluding the first number of the series and including the next number. This operation generates a new subset of numbers, which is averaged. This process is repeated over the entire series.

The acronyms "Lowess" and "Loess" are derived from the term "Locally weighted scatter plot smoothing" as both methods are based on locally weighted linear regression to smooth the data. The linear regression is performed over a limited number of points called the span. The smoothing process is therefore local because each smoothed value is determined only by neighbouring data points, the ones within the span. The process is weighted because a regression weight function is used to fit the data points contained within the span. The data point to be smoothed has the largest weight and the most influence on the fit. The more distant the points from the one to be fitted, the lower their weight and points outside the span are given zero weight and do not influence the fit. The two approaches are similar but differ in the model used for the regression: Lowess implements a linear polynomial, while Loess implements a quadratic polynomial.

Rlowess and Rloess are robust versions of Lowess and Loess, to reduce the sensitivity to outliers. Robustness is achieved by an appropriate choice of the weight function. These robust methods include an additional calculation of robust weights, based on MAD, which is resistant to outliers.

The Savitzky-Golay filter can be considered a generalisation of the Loess approach allowing to choose higher order polynomials for the fit. In the applications presented in this paper, the degree of the polynomial is always 6 or higher.

To assess the comparative performance of SVR, a wide range of functions have been tested, ranging from power laws to exponential, trigonometric and squashing functions. For brevity of explanation, in the following the attention will be focused on two quite difficult types of functions:

$$f_1 = \sin(\sqrt{t}) + \cos(t) \qquad (9)$$

$$f_2 = \sin(3t) + \cos(t^2) \cdot \exp\left(-\frac{t}{6}\right) \qquad (10)$$
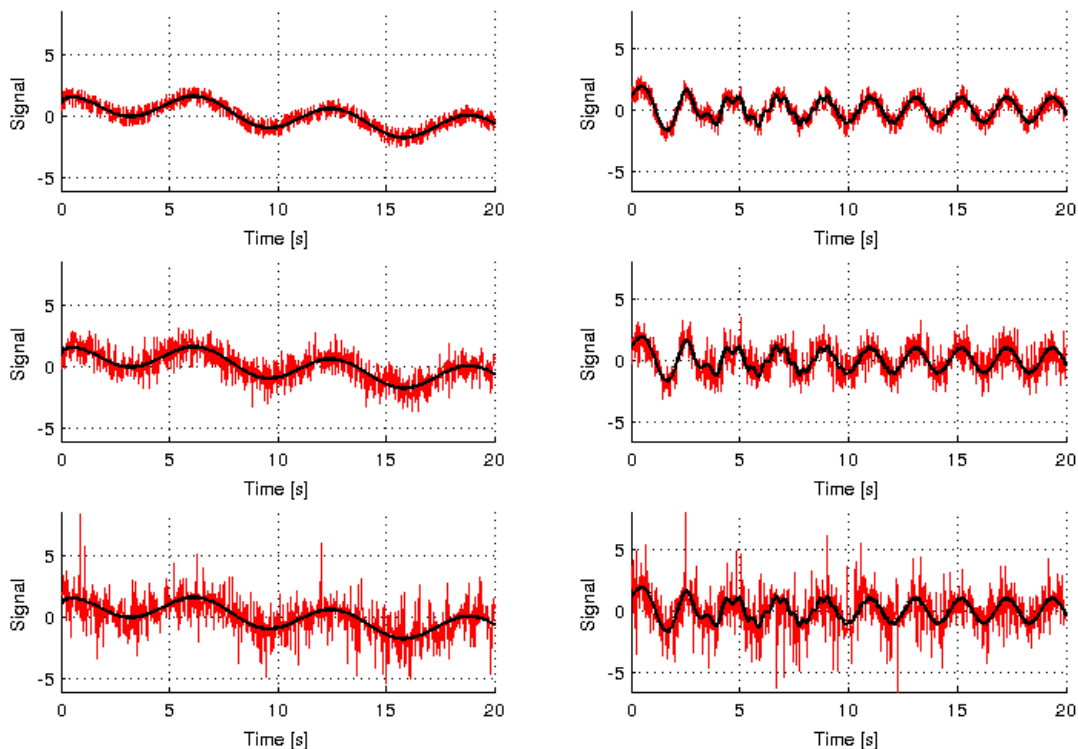

Figure 4 Left: realization of function f_1 including noise Right: realization of function f_2 including noise. Top rows: random noise. Central rows: Gaussian Noise. Bottom rows: noise with a Laplacian distribution.

To the synthetic data generated for these functions a variety of noise types, noise amplitudes and outliers have been added. The main systematic tests performed have involved the following types of uncertainties:

- Random noise in the range from $-0.5\ max(|y_{data}|)$ to $+0.5\ max(|y_{data}|)$
- Gaussian Noise with Standard deviation of $0.5\ max(|y_{data}|)$
- Noise with a Laplacian pdf with parameter b equal to $0.5\ max(|y_{data}|)$
- Outliers: to simulate outliers, 10% of the points have been derived by a second Gaussian distribution with standard deviation equal to $0.2\ max(|y_{data}|)$

In Figure 4, these two functions are plotted, showing their trends and the level of added noise.

In general, the SVR tends to perform not worse and very often better than all the other methods. Particularly relevant is the robustness against outliers. This can be appreciated from the plots of Figure 5, in which the RMSE (Root Main Square Error) of equations (9) and (10) are reported for the case of outliers generated as described above. The RMSE is calculated on the basis of the differences between the curves, smoothed with the various methods, and the exact, analytical functions used to generated the data. Three different databases, representing cases of different amounts of data available, have been considered: datasets of 500 points, 1000 points and 2000 points.
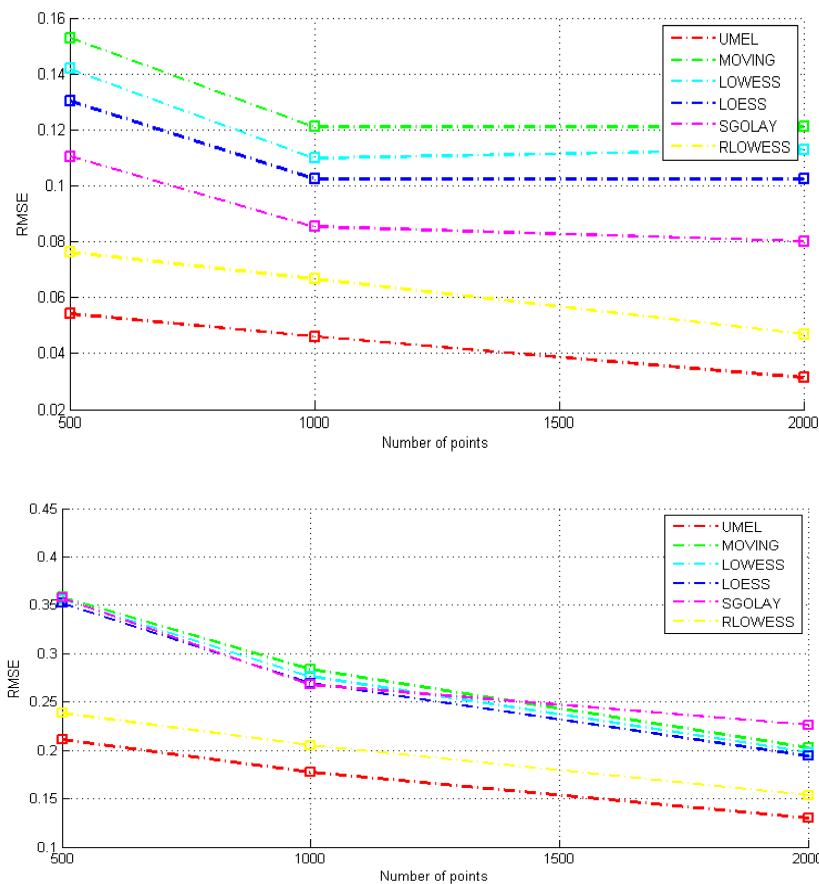

Figure 5 Robustness against outliers quantified as RMSE with respect to the exact formula. Top: results for equation (9) and datasets of 500, 1000 and 2000 entries. Bottom: results for equation (10) and datasets of 500, 1000 and 2000 entries. SVR provides always the best results, i.e. the lowest RMSE.

4. THE LIDAR SYSTEM

The measurements described in the paper have been performed with the mobile Lidar unit designed and developed at Industrial Engineering Department, University of Rome "Tor Vergata" [8, 14]. The system consists of an easily transportable compact Lidar system. The transmitter is a Nd:YAG laser that can operate at three wavelengths: 1064, 532 and 355nm. Of these wavelengths, the 532 nm is not in the eye safe region of the spectrum. The other two, on the contrary, are currently used for monitoring the atmosphere and for surveying even populated areas. The laser is anchored at the receiver system, a Newtonian telescope, and both can move to cover a whole hemisphere. The system is completely auto-powered and the structure is designed to be transportable and steerable. It is easily hooked to azimuth mount for supporting and rotating about two mutually perpendicular axes; one vertical, from -10° to 90°, and one horizontal, from 0° to 220°. Two step-motors provide a global angular resolution of 1.8°. Since the laser source is operating in the UV region, the detector chosen is a Hamamatsu's photomultiplier tube (PMT), R3235 model. These technologies have become relatively standard and therefore they can be procured at reasonable costs [18]. The main characteristics of the mobile unit are reported in Table 1. The entire apparatus is controlled by a software package, written in Labview and Matlab, explicitly developed for this application. The laser activation and the wavelength selection, together with the rotation of the telescope and data acquisition, is controlled by a Labview series of routines. The signal processing algorithms and the visualization of the results have been implemented using Matlab.

The signals analysed in this paper have been collected during an extensive experimental campaign, which has been carried out in Calabria, in the south of Italy.

Table 1. Parameters of Nd:Yag Lidar system [13].

Transmitter:	
Laser	Q-switch Nd:Yag
Energy pulse at 1064 nm	360 mJ
Pulse time width	5 ns
Divergence angle	0,5 mrad
Pulse Frequency	10 Hz
Receiver:	
Telescope type	Newtonian
Nominal focal length	1030 mm
Primary mirror diameter	210 mm
Detector	Photomultiplier (PMT)
Photocathode sensibility	72 mA/W
Response time	30 ns

5. ANALYSIS OF EXPERIMENTAL DATA

The potential of SVR for smoothing and filtering of signals has been shown quite clearly with the numerical tests and studies describe in Section 2. Another competitive advantage of Support Vector Regression consists of the fact that its parameters can be chosen without detailed knowledge of the noise superimposed on the signal. Only the amplitude of the noise really matters, because typically it affects the choice of the e-tube. Moreover this filtering technique is quite robust to variations in the details of the noise as has already been demonstrated in other applications of LIDAR detection. [10, 12, 13]. In the case of detection of widespread smoke, the most innovative application described in the paper, the quality of the signals after filtering can be appreciated in Figure 6. At this point, it is relatively easy to perform a fitting of the signals and, when the maximum of the signal is above a certain threshold, a warning of widespread smoke can be issued.
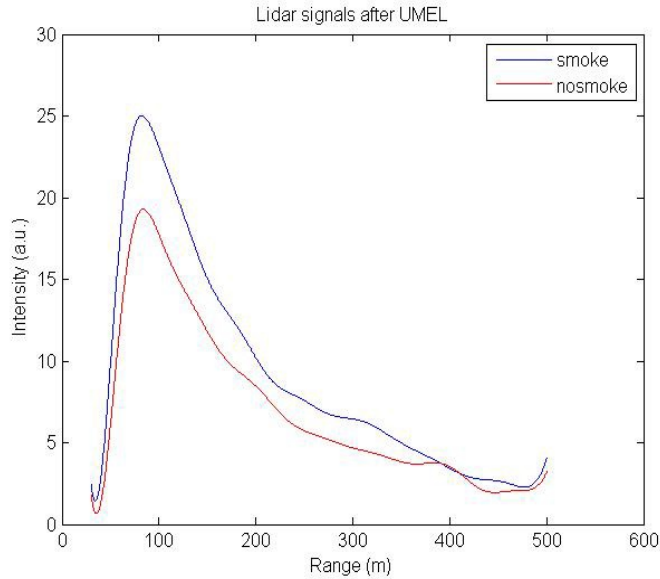
Figure 6. The experimental signals of Figure 1 after filtering with SVR. The case without smoke is in red. The signal for the case with widespread smoke is in blue.

Starting from the typical Lidar equation [8], it has been decided to fit the decaying part of the backscattered signal intensity with a mathematical expression of the form:

$$P = \frac{K_1}{R^2} \exp(-2K_2 R) \qquad (11)$$

where K_1 and K_2 are constants and R is the range. The data of Figure 3 have been fitted with this formula. The results of the nonlinear fit are:

- In case of widespread smoke:

$$P = \frac{2.648 \cdot 10^{-1}}{R^2} \cdot \exp(-1.259 \cdot 10^{-3} \cdot R)$$

$$(12)$$

- In the case of no smoke:

$$P = \frac{1.734 \cdot 10^{-1}}{R^2} \cdot \exp(-1.171 \cdot 10^{-3} \cdot R)$$

$$(13)$$

The results of the fit, equations (12) and (13), indicate quite clearly that the parameter K_2 is practically the same for both the case of widespread smoke and clear atmosphere. On the other hand, there is a significant difference, of the order of 25%, in the constants K_1. This is expected since K_1 includes the effect of the coefficient β, which indeed quantifies the backscattering properties of the atmosphere [5].

The results detailed in the previous fits have been confirmed by a first statistical analysis of a set of laser pulses. The results are shown in the plot of Figure 7, in which the K_1 values of 50 signals with widespread smoke and 50 signals from clear atmosphere are reported. By simple inspection, it can be seen how a simple threshold in the coefficient K_1 can discriminate between the cases of widespread smoke and not smoke. Indeed in the case of the signals analyzed to produce Figure 7, with this simple approach a success rate of almost 95% can be achieved.
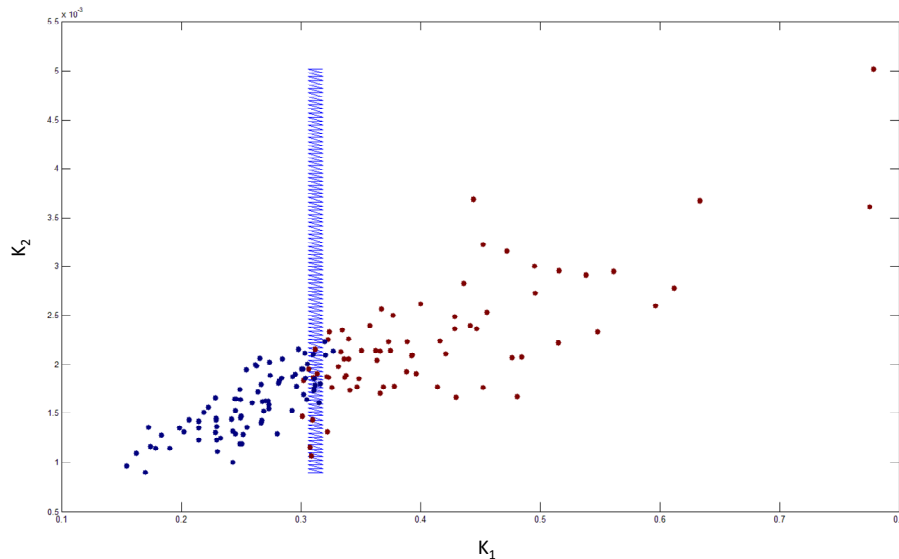
Figure 7. Values of K_1 for 50 pulses in clear atmosphere and 50 pulses with widespread smoke. Blue points: measurements obtained in clear atmosphere. Red points: measurements obtained in presence of wide spread smoke. The vertical band manages to separate the two cases with almost 95% of success rate. K_2 is reported on the vertical axis K_1 on the horizontal.

6 CONCLUSIONS AND FUTURE DEVELOPMENTS

As shown in the previous section, SVR is a sophisticated technique to filter the backscattered signals of LIDAR systems. If this filtering step is successful, it is relatively easy to fit the signals and calculate the parameters K_1 and K_2. Based on these values, a decision can be made about the presence of widespread smoke. The effectiveness of the approach will therefore depend on the accuracy and reliability of the first filtering step. The first results are encouraging but a wider statistical study is necessary. In this perspective, more examples of widespread smoke will have to be collected. It is expected that for more complicated cases both parameters K_1 and K_2 will have to be used to discriminate between propagation in clear atmosphere and widespread smoke. Classifiers based on SVM are being developed explicitly for this purpose. Also, in practical applications, the power output of the laser will have to be monitored to make sure that the changes in the amplitude of the received signals are really due to variations in the atmosphere and not drifts in the system.

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